

Processing 2 jobs through k machines

Step-1

Draw two perpendicular lines, horizontal one representing the processing time for job 1 while job 2 remains idle and the vertical one representing the processing time for job 2 while job 1 remains idle.

Step-2

Mark the processing time for jobs 1 and 2 on the horizontal and vertical lines respectively according to the given order of machines.

Step-3

Construct various blocks starting from the origin (starting point) by passing the same machines until the end point.

Step-4

Draw the line starting from origin to end point by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal line (base). The horizontal segment of this line indicates that first job is under process while second job is idle. Similarly, the vertical

segment of the line indicates that the second job is under process while first job is idle. The diagonal segment of the line shows that both the jobs are under process simultaneously.

Step-5

An optimum path is one that minimizes the idle time for both the jobs, thus, we must choose the path on which diagonal movement is maximum.

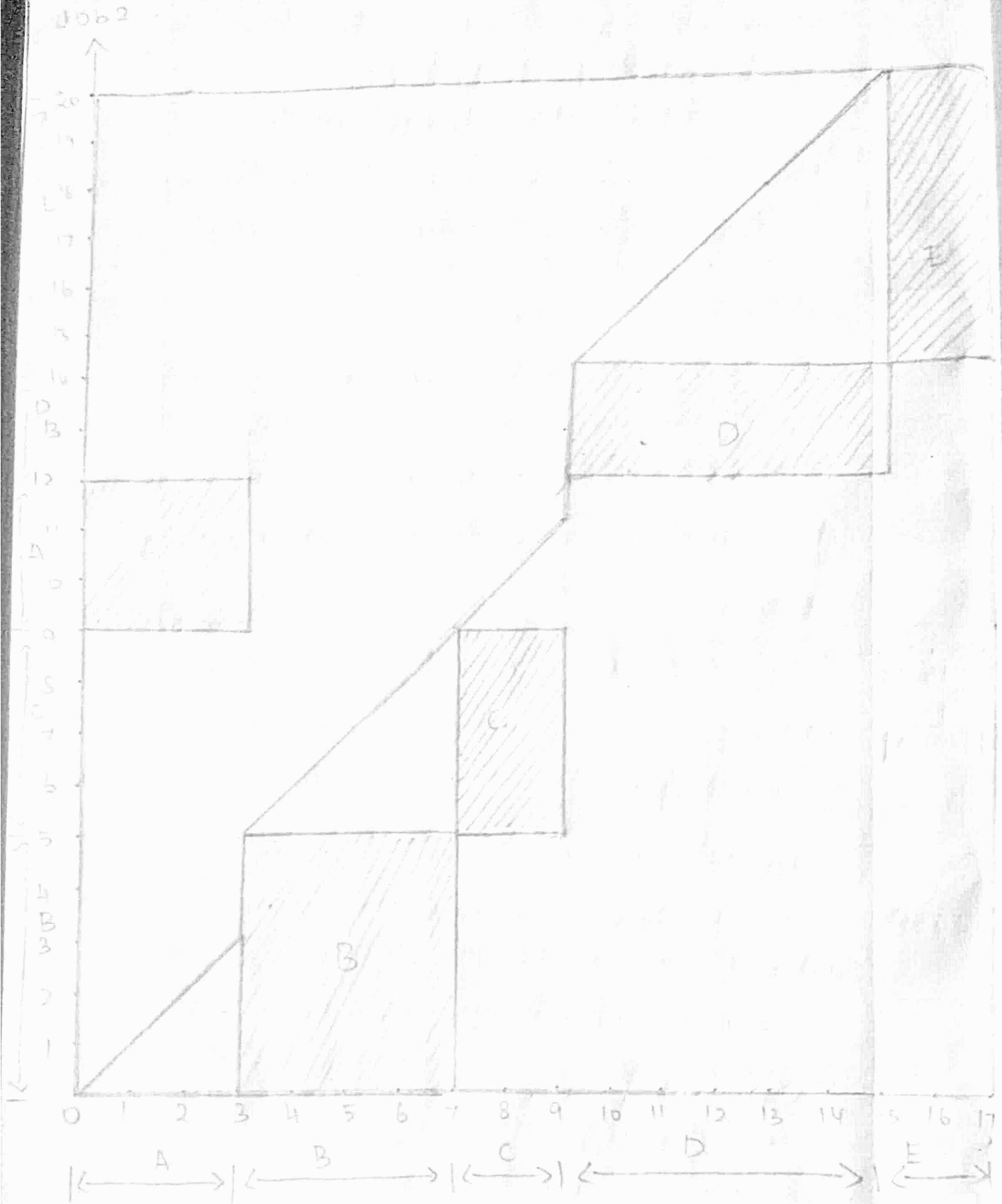
Step-6

The total elapsed time is obtained by adding the idle time for either job to the processing time for that job.

Sample problems.

1221) Use graphical method to minimize the time added to process the following jobs on the machines shown i.e. for each machine find the job which should be done first. Also calculate the total time elapsed to complete both the jobs.

Job 1	{	sequence	A	B	C	D	E
		Time	3	4	2	6	2
Job 2	{	sequence	B	C	A	D	E
		Time	5	4	3	2	6



Total elapsed time for Job 1 = Processing time of Job 1 + Idle time for Job 1

$$= 17 + 5 = 22 \text{ hours}$$

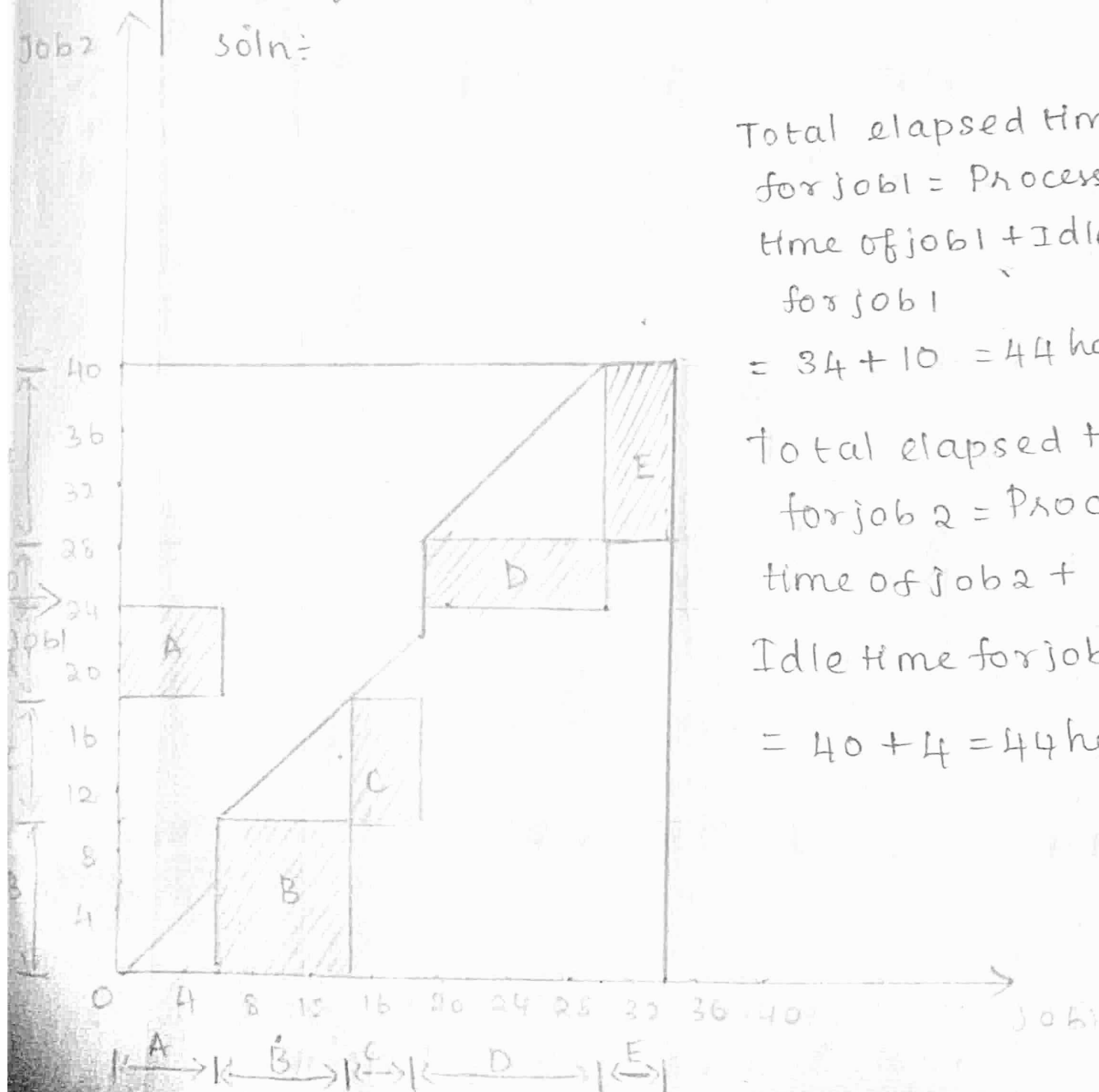
Total elapsed time for Job 2 = Processing time of Job 2 + Idle time for Job 2

$$= 20 + 2 = 22 \text{ hours}$$

1222) Using graphical method. Calculate the minimum time needed to process job 1 and 2 machines A, B, C, D and E (ie) for each machine find the job which should be done first calculate the total time needed to complete both jobs.

		Machines					
Job 1	{	sequence:	A	B	C	D	E
		Time(hours):	6	8	4	12	4
Job 2	{	sequence:	B	C	A	D	E
		Time(hours):	10	8	6	4	12

Soln:



Total elapsed time for job 1 = Processing time of job 1 + Idle time for job 1

$$= 34 + 10 = 44 \text{ hours}$$

Total elapsed time for job 2 = Processing time of job 2 + Idle time for job 2

$$= 40 + 4 = 44 \text{ hours}$$

Processing n jobs through 2 machines optimum sequence Algorithm

step-1

List the jobs along with their processing times in a table as shown below.

Job number :	J_1	J_2	J_3	$\dots$	J_n
Processing time on machine M_1 :	t_{11}	t_{12}	t_{13}	$\dots$	t_{1n}
on machine M_2 :	t_{21}	t_{22}	t_{23}	$\dots$	t_{2n}

step-2

Examine the rows for processing times on machines M_1 and M_2 , and find the smallest processing time in each row, i.e. find out $\min(t_{1j}, t_{2j})$ for all j .

step-3

If the smallest processing time is for the first machine M_1 , then place the corresponding job in the first available position in the sequence. If it is for the second machine, then place the corresponding job in the last available position in the sequence.

step-4

If there is a tie in selecting the minimum of all the processing times, then

there may be three situations:

(a) Minimum among all processing times is same for the machine i.e., $\min(t_{1j}, t_{2j}) = t_{1k} = t_{2r}$, then process the k th job first and the r th job last.

(c) If the tie for the minimum occurs among processing times t_{2j} on machine M_2 , then select arbitrarily the job to process last.

Step-5

Cross off the jobs already assigned and repeat steps 1 through 4, placing the remaining jobs next to first or next to last, until all the jobs have been assigned.

Step-6

calculate idle time for machines M_1 and M_2 .

(a) Idle time for M_1 = Total elapsed time
- (Time when the last job in a sequence finishes on M_1)

(b) Idle time for M_2 = time at which the first job in a sequence finishes on M_1 + $\sum_{j=2}^n \{ (\text{Time when the } j^{\text{th}} \text{ job in a sequence starts on } M_2) - (\text{Time when the } (j-1)^{\text{th}} \text{ job in a sequence finishes on } M_2) \}$.

Step-7

The total elapsed time to process all jobs through two machines is as under:

Total elapsed time = Time when the n^{th} job in a sequence finishes on machine M_2 .

$$= \sum_{j=1}^n t_{2j} + \sum_{j=1}^n I_{2j}$$

where t_{2j} = Time required for processing j^{th} job on machine M_2 .

I_{2j} = Time for which machine M_2 remains idle after processing $(j-1)^{\text{th}}$ job and before starting work on j^{th} job.

Problems

12017 In a factory, there are six jobs to perform each of which should go through two machines A and B, in the order A, B. The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the

Jobs that would minimize the total elapsed time, T . What is the value of T ?

Job :	J_1	J_2	J_3	J_4	J_5	J_6
Machine A :	1	3	8	5	6	3
Machine B :	5	6	3	2	2	10

Soln:

Job	J_1	J_2	J_3	J_4	J_5	J_6
Machine A	1	3	8	5	6	3
Machine B	5	6	3	2	2	10

J_1	J_6	J_2	J_3	J_4	J_5
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Job	Machine A		Machine B		Idle Time for Machine B
	In	Out	In	Out	
J_1	0	1	1	6	1
J_6	1	4	6	16	0
J_2	4	7	16	22	0
J_3	7	15	22	25	0
J_4	15	20	25	27	0
J_5	20	26	27	29	0

Total elapsed time = 29 hours

Idle time for machine A = $29 - 26 = 3$ hours

Idle time for machine B = 1 hour

1202)

A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The time required to perform the printing and binding operations for each book is shown below. Determine the order in which books should be processed, in order to minimize the total time required to turn out all the books.

Book	1	2	3	4	5	6
Printing Time (hrs)	30	120	50	20	90	100
Binding Time (hrs)	80	100	90	60	30	10

Soln:-

Book	1	2	3	4	5	6
Printing Time (hrs)	30	120	50	20	90	100
Binding Time (hrs)	80	100	90	60	30	10

4	1	3	2	5	6
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Book	Printing machine		Binding machine		Idle time of binding machine
	Time in	Time out	Time in	Time out	
4	0	20	20	80	20
1	20	50	80	160	0
3	50	100	160	250	0
2	100	220	250	350	0
5	220	310	350	380	0
6	310	410	410	420	30

Total elapsed time = 420 hours

Idle time for ^{Printing} Machine A = $420 - 410 = 10$ hours

Idle time for binding machine = $20 + 30 = 50$ hours

1203) We have five jobs, each of which must go through the two machines A and B in the order AB. Processing times in hours are given in the table below:

Job (i):	1	2	3	4	5
Machine A (A _i)	5	1	9	3	10
Machine B (B _i)	2	6	7	8	4

Determine a sequence for the five jobs

That will minimize the elapsed time.
soln:

Job (i) : 1 2 3 4 5
Machine A : ~~5~~ 1 9 3 10
(A_i)
Machine B : 2 ~~6~~ 7 ~~8~~ 4
(B_i)

2	4	3	5	1
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Job (i)	Machine A (A _i)		Machine B (B _i)		Idle Time for machine B (B _i)
	In	Out	In	Out	
2	0	1	1	7	1
4	1	4	7	15	0
3	4	13	15	22	0
5	13	23	23	27	1
1	23	28	28	30	1

Total elapsed time = 30 hours

Idle time for Machine A (A_i) = 2 hours

Idle time for Machine B (B_i) = 3 hours

1204) Six jobs go first over machine I and then machine II. The order of the completion of jobs has no significance. The following table gives the machine times in hours

for six jobs and the two machines.

Job No:	1	2	3	4	5	6
Time on						
Machine I (A _i)	5	9	4	7	8	6
Time on						
Machine II (B _i)	7	4	8	3	9	5

Find the sequence of jobs that minimizes the total elapsed time to complete the jobs. Find the minimum time by using Gantt's chart or by any other method. Also compute the idle time for both the machines.

Soln:

Job No:	1	2	3	4	5	6
Time on						
Machine I (A _i)	5	9	4	7	8	6
Time on						
Machine II (B _i)	7	4	8	3	9	5

3	1	5	6	2	4
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Job No	Time on Machine I (A _i)		Time on Machine II (B _i)		Idle Time for Machine II (B _i)
	In	Out	In	Out	
3	0	4	4	12	4
1	4	9	12	19	0
5	9	17	19	28	0
6	17	23	28	33	0
2	23	32	33	37	0
4	32	39	39	42	2

Total elapsed time = 42 hours

Idle time for Machine I (A_i) = 42 - 39 = 3 hours

Idle time for machine II (B_i) = 6 hours.

(205) A company has six jobs on hand coded 'A' to 'F'. All the jobs have to go through two machines 'M I' and 'M II'. The time required for each job on each machine in hours is given below.

	A	B	C	D	E	F
M I	3	12	18	9	15	6
M II	9	18	24	24	3	15

Draw a sequence table scheduling the six jobs on the two machines.

	A	B	C	D	E	F
M I	3	12	18	9	15	6
M II	9	18	24	24	3	15

A	F	D	B	C	E
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Job	M-I		M-II		Idle time for M-II
	In	Out	In	Out	
A	0	3	3	12	3
F	3	9	12	27	0
D	9	18	27	51	0
B	18	30	51	69	0
C	30	48	69	93	0
E	48	63	93	96	0

Total elapsed time = 96 hours

Idle time for M-I = $96 - 63 = 33$ hours

Idle time for M-II = 3 hours

1206) Seven jobs are to be processed on two machines A and B in the order A → B. Each machine can process only one job at a time.

The processing times (in hours) are as follows.

Job	1	2	3	4	5	6	7
Machine A	10	12	13	7	14	5	16
Machine B	15	11	8	9	6	7	16

Suggest optimum sequence of processing the jobs and the total elapsed time.

Soln:

Job	1	2	3	4	5	6	7
machine A	10	12	13	7	14	5	16
machine B	15	11	8	9	6	7	16

6	4	1	7	2	3	5
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Job	machine A		Machine B		Idle time for Machine B
	In	Out	In	Out	
6	0	5	5	12	5
4	5	12	12	21	0
1	12	22	22	37	1
7	22	38	38	54	1
2	38	50	54	65	0
3	50	63	65	73	0
5	63	77	77	83	4

Total elapsed time = 83 hours

Idle time for Machine A = $83 - 77 = 6$ hours

Idle time for Machine B = 11 hours.

1207) In the machine shop, 8 different products are being manufactured each requiring time on two machines A and B as given below

Product	Time (in min) on machine A	Time (in min) on machine B
<u>I</u>	30	20
<u>II</u>	45	30
<u>III</u>	15	50
<u>IV</u>	20	35
<u>V</u>	80	36
<u>VI</u>	120	40
<u>VII</u>	65	50
<u>VIII</u>	10	20

Decide the optimum sequence of processing of different products in order to minimize the total manufacturing time for all the products. Name and discuss the scheduling model used.

Soln:

Product	<u>I</u>	<u>II</u>	<u>III</u>	<u>IV</u>	<u>V</u>	<u>VI</u>	<u>VII</u>
Machine A	30	45	15	20	80	120	65
Machine B	20	30	50	35	36	40	50

<u>VIII</u>	<u>III</u>	<u>IV</u>	<u>VII</u>	<u>VI</u>	<u>V</u>	<u>II</u>	<u>I</u>
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Product	Machine A		Machine B		Idle time for Machine B
	In	Out	In	Out	
VIII	0	10	10	30	10
III	10	25	30	80	0
IV	25	45	80	115	0
VII	45	110	115	165	0
VI	110	230	230	270	65
V	230	310	310	346	40
II	310	355	355	385	9
I	355	385	385	405	0

Total elapsed time = 6 hours 45 minutes

Idle time for Machine A = $405 - 385$

= 20 minutes

Idle time for Machine B = $10 + 65 + 40 + 9$

= 124 minutes

= 2 hours 4 minutes

(208) The maintenance crew of a company is divided into two parts C_1 and C_2 which cares for the maintenance of the seven machines $M_1, M_2, \dots, M_7$ in the factory. Crew C_1 is responsible for replacement of the parts which are worn out while crew C_2 oils and resets the machines

back for operation. The time required by Crews C_1 and C_2 on different machines is given below.

Machines	Crew C_1	Crew C_2
M_1	24	10
M_2	14	20
M_3	22	8
M_4	20	18
M_5	30	10
M_6	28	6
M_7	16	4

In what order the machines should be handled by Crew C_1 and C_2 so that the total time taken is minimized. Also determine what shall this time be?

Soln:

Machines	M_1	M_2	M_3	M_4	M_5	M_6	M_7
Crew C_1	24	14	22	20	30	28	16
Crew C_2	10	20	8	18	10	6	4

M_2	M_4	M_1	M_5	M_3	M_6	M_7
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Machines	Crew C ₁		Crew C ₂		Idle time for Crew C ₂
	In	Out	In	Out	
M ₂	0	14	14	34	14
M ₄	14	34	34	52	0
M ₁	34	58	58	68	6
M ₅	58	88	88	98	20
M ₃	88	110	110	118	12
M ₆	110	138	138	144	20
M ₇	138	154	154	158	10

Total elapsed time = 158 hours

Idle time for Crew C₁ = 158 - 154
= 4 hours

Idle time for Crew C₂ = 82 hours

Processing n jobs through k machines Optimal Sequence Algorithm

Step-1

Find $\min t_{1j}$, $\min t_{kj}$ and maximum each of

$$t_{2j}, t_{3j}, \dots, t_{k-1j} \quad \forall j = 1, 2, \dots, n$$

Step-2

Check the following

- (a) $\min t_{1j} \geq \max t_{ij}, \quad \forall i = 2, 3, \dots, k-1$
(b) $\min t_{kj} \geq \max t_{ij}, \quad \forall i = 2, 3, \dots, k-1$

Step-3

If the inequalities of step 2 are not satisfied, method fails. Otherwise go to next step.

Step-4

Convert the k machine problem into a two-machine problem by introducing two fictitious machines G and H such that

$$t_{Gj} = t_{1j} + t_{2j} + \dots + t_{k-1j}$$

$$\text{and } t_{Hj} = t_{2j} + t_{3j} + \dots + t_{kj}$$

Step-5

Determine the optimal sequence for the ' n ' jobs and ' 2 ' machines equivalent

sequencing problem with the prescribed order of jobs in the same way as discussed earlier. The resulting sequence shall be optimum for the given problem.

Sample problems:

209) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information processing time on machines is given in hours and passing is not allowed.

Job	A	B	C	D	E	F	G
Machine M_1	3	8	7	4	9	8	7
Machine M_2	4	3	2	5	1	4	3
Machine M_3	6	7	5	11	5	6	12

Soln:

$$\text{Min of } M_1 = 3$$

$$\text{Min of } M_3 = 5$$

$$\text{Max of } M_2 = 5$$

$$\text{Min}(M_3) \geq \text{max of } (M_2)$$

$$G = M_1 + M_2 \quad / \quad H = M_2 + M_3$$

Job	A	B	C	D	E	F	G
G	7	11	9	9	10	12	10
H	16	10	7	16	6	10	15

A	D	G	B	F	C	E
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Job	M ₁		M ₂		M ₃		Idle time for M ₂	Idle time for M ₃
	In	Out	In	Out	In	Out		
A	0	3	3	7	7	13	3	7
D	3	7	7	12	13	24	0	0
G	7	14	14	17	24	36	2	0
B	14	22	22	25	36	43	5	0
F	22	30	30	34	43	49	5	0
C	30	37	37	39	49	54	3	0
E	37	46	46	47	54	59	7	0

Total elapsed time = 59 hours

Idle time for Machine M₁ = 59 - 46 = 13 hours

Idle time for Machine M₂ = 59 - 47 + 25
= 37 hours

Idle time for machine M₃ = 7 hours

1210) We have 4 jobs each of which has to go through the machines M_j (j = 1, 2, ..., 6) in order M₁, M₂, ..., M₆. Processing time (in hours) is given below:

	Machines					
	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆
Job A :	18	8	7	2	10	25
Job B :	17	6	9	6	8	19
Job C :	11	5	8	5	7	15
Job D :	20	4	3	4	8	12

Determine a sequence of these four jobs that minimizes the total elapsed time.

Soln:

Min of $M_1 = 11$, max of $(M_2, M_3, M_4, M_5) = 10$
 min of $M_6 = 12$

Min of $M_1 \geq \max$ of (M_2, M_3, M_4, M_5)

min of $M_6 \geq \max$ of (M_2, M_3, M_4, M_5)

$$G = M_1 + M_2 + M_3 + M_4 + M_5$$

$$H = M_2 + M_3 + M_4 + M_5 + M_6$$

Jobs	A	B	C	D
G	45	46	36	39
H	52	48	40	31

C	A	B	D
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Total elapsed time = 112 hours

Idle time for $M_1 = 112 - 66 = 46$ hours

Idle time for $M_2 = 112 - 70 + 47 = 89$ hours

Idle time for $M_3 = 112 - 73 + 46 = 85$ hours

Idle time for $M_4 = 112 - 77 + 59 = 94$ hours

Idle time for $M_5 = 112 - 85 + 52 = 79$ hours

Idle time for $M_6 = 41$ hours.

1211)

Solve the following sequencing problem when passing out is not allowed.

Item	Machine (Processing time in hours)			
	A	B	C	D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

Soln:-

$$\text{Min of (A)} = 12$$

$$\text{Min of (D)} = 12$$

$$\text{Max of (B, C)} = 10$$

$$\text{Min of (A)} \geq \text{Max of (B, C)}$$

$$\text{Min of (D)} \geq \text{Max of (B, C)}$$

$$G = A + B + C \quad ; \quad H = B + C + D$$

Item	I	II	III	IV
Machine G	24	24	24	24
Machine H	24	24	24	24

I	II	III	IV
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Item	M-A		M-B		M-C		M-D		Idle time for M-B	Idle time for M-C	Idle time for M-D
	In	Out	In	Out	In	Out	In	Out			
I	0	15	15	20	20	24	24	39	15	20	24
II	15	27	27	29	29	39	39	51	7	5	0
III	27	43	43	46	46	51	51	67	14	7	0
IV	43	60	60	63	63	67	67	84	14	12	0

Total elapsed time = 84 hours

Idle time for Machine A = $84 - 60 = 24$ hours

Idle time for Machine B = $84 - 63 + 50 = 71$ hours

Idle time for Machine C = $84 - 67 + 44 = 61$ hours

Idle time for Machine D = 24 hours

Exercise problems.

1212)

Given the following data

a) Job :	1	2	3	4	5	6
Machine A:	12	10	9	14	7	9
Machine B:	7	6	6	5	4	4
Machine C:	6	5	6	4	2	4

(b) Order of processing jobs : A C B

c) ~~the~~ sequence suggested : Jobs 5, 3, 6, 2, 1, 4

d) Determine the total elapsed time for the sequence suggested?

(i) Is the given sequence optimal?

(ii) If your answer is 'No' determine the optimal sequence and the total elapsed

Time associated with it

soln:

(i) 73 hours

(ii) No

(iii) Min of A = 7

Min of C = 2

Max of B = 7

Min of A $\geq$ Max of B

$G = A + B$; $H = B + C$

Job	1	2	3	4	5	6
G	19	16	15	19	11	13
H	13	11	12	9	6	8

1	3	2	4	6	5
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Job	M-A		M-B		M-C		Idle time for Machine B	Idle time for Machine C
	In	Out	In	Out	In	Out		
1	0	12	12	19	19	25	12	19
3	12	21	21	27	27	33	2	2
2	21	31	31	37	37	42	4	4
4	31	45	45	50	50	54	8	8
6	45	54	54	58	58	62	4	4
5	54	61	61	65	65	67	3	3

Total elapsed time = 67 hours

Idle time for Machine A = $67 - 61 = 6$ hours

Idle time for Machine B = $67 - 61 + 33 = 39$ hours

Idle time for Machine C = 40 hours

- (215) Find the sequence that minimizes the total time required in performing the following jobs, on three machines in the order ABC.

Processing Time (in hours) on	Job					
	1	2	3	4	5	6
Machine A	8	3	7	2	5	1
Machine B	3	4	5	2	1	6
Machine C	8	7	6	9	10	9

Soln:

Min of M-A = 1

Min of M-C = 6

Max of M-B = 6

Min of C $\geq$ Max of B

$G = A + B$, $H = B + C$

Job	1	2	3	4	5	6
G	11	7	12	4	6	7
H	11	11	11	11	11	15

4	5	6	2	1	3
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Job	M-A		M-B		M-C		Idle time for M-B	Idle time for M-C
	In	Out	In	Out	In	Out		
A	0	2	2	4	4	13	2	4
5	2	7	7	8	13	23	3	0
6	7	8	8	14	23	32	0	0
2	8	11	14	18	32	39	0	0
1	11	19	19	22	39	47	1	0
3	19	26	26	31	47	53	4	0

Total elapsed time = 53 hours

Idle time for M-A = $53 - 26 = 27$ hours

Idle time for M-B = $53 - 31 + 10 = 32$ hours

Idle time for M-C = 4 hours.

1213) We have five jobs, each of which must go through machines A, B and C in the order A B C. Processing times (in hours) are given in the following table:

Job	1	2	3	4	5
Machine A. (A _i)	8	10	6	7	11
Machine B. (B _i)	5	6	2	3	4
Machine C. (C _i)	4	9	8	6	5

Min of A = 6

Min of C = 4

Max of B = 6

Min of A $\geq$ Max of B

G = A + B ; H = B + C

Job	1	2	3	4	5
G	13	16	8	10	15
H	9	15	10	9	9

3	2	4	1	5
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Job	M-A		M-B		M-C		Idle time for M-B	Idle time for M-C
	In	Out	In	Out	In	Out		
3	0	6	6	8	8	16	6	8
2	6	16	16	22	22	31	8	6
4	16	23	23	26	31	37	1	0
1	23	31	31	36	37	41	5	0
5	31	42	42	46	46	51	6	5

Total elapsed time = 51 hours

Idle time for Machine A = 51 - 42 = 9 hours

Idle time for Machine B = 51 - 46 + 26 =

Idle time for Machine C = 19 hours

1214) Determine the optimal sequence of jobs that minimizes the total elapsed time

based on the following information processing
Time on machines given in hours and passing
is not allowed.

Job :	1	2	3	4	5
Machine A :	3	8	7	5	2
Machine B :	3	4	2	1	5
Machine C :	5	8	10	7	6

soln:

$$\text{Min of A} = 2$$

$$\text{Min of C} = 5$$

$$\text{Max of B} = 5$$

$$\text{Min of C} \geq \text{Max of B}$$

$$G = A + B; H = B + C$$

Job	1	2	3	4	5
G	6	12	9	6	7
H	8	12	12	8	11

1	4	5	3	2
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Job	M-A		M-B		M-C		Idle time for M-B	Idle time for M-C
	In	Out	In	Out	In	Out		
1	0	3	3	6	6	11	3	6
4	3	8	8	9	11	18	2	0
5	8	10	10	15	18	24	1	0
3	10	17	17	19	24	34	2	0
2	17	25	25	29	34	42	6	0

Total elapsed time = 42 hours

Idle time for Machine A = $42 - 25 = 17$ hours

Idle time for Machine B = $42 - 29 + 14$
= 27 hours

Idle time for Machine C = 6 hours

1218) Solve the following sequencing problem, giving an optimal solution when passing is not allowed.

	Job				
	A	B	C	D	E
Machine M_1	10	12	8	15	16
Machine M_2	3	2	4	1	5
Machine M_3	5	6	4	7	3
Machine M_4	14	7	12	8	10

Soln:

Min of machine $M_1 = 8$

Min of machine $M_4 = 7$

Max of machine $(M_2, M_3) = 7$

Min of machine $M_1 \geq$ Max of machine M_2

Min of machine $M_4 \geq$ Max of machine M_3

$$G = M_1 + M_2 + M_3, \quad H = M_2 + M_3 + M_4$$

Job	A	B	C	D	E
G	18	20	16	23	24
H	22	15	20	16	18

C	A	E	D	B
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Job	M-1		M-2		M-3		M-4		Idle time for M-2	Idle time for M-3	Idle time for M-4
	In	Out	In	Out	In	Out	In	Out			
C	0	8	8	12	12	16	16	28	8	12	16
A	8	18	18	21	21	26	28	42	6	5	0
E	18	34	34	39	39	42	42	52	13	13	0
D	34	49	49	50	50	57	57	65	10	8	5
B	49	61	61	63	63	69	69	76	11	6	4

Total elapsed time = 76 hours

Idle Time for Machine I = $76 - 61 = 15$ hours

Idle Time for Machine II = $76 - 63 + 48 = 61$ hours

Idle Time for Machine III = $76 - 69 + 44 = 51$ hours

Idle Time for Machine IV = 25 hours

(219) Solve the following sequencing problem, giving an optimal solution when passing

is not allowed, machine (processing time in hours)

Job	M ₁	M ₂	M ₃	M ₄
1	20	3	3	25
2	12	5	1	11
3	18	4	2	10
4	17	2	4	28

$$\text{Min of } M_1 = 12$$

$$\text{Min of } M_4 = 10$$

$$\text{Max of } (M_2, M_3) = 5$$

$$\text{Min of } M_1 \geq \text{Max of } (M_2, M_3)$$

$$\text{Min of } M_4 \geq \text{Max of } (M_2, M_3)$$

$$G = M_1 + M_2 + M_3 ; H = M_2 + M_3 + M_4$$

Job 1 2 3 4

G 26 18 24 23

H 31 17 16 34

4	1	2	3
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Job	M-1		M-2		M-3		M-4		Idle Time for M-2	Idle Time for M-3	Idle Time for M-4
	In	Out	In	Out	In	Out	In	Out			
4	0	17	17	19	19	23	23	51	17	19	23
1	17	37	37	40	40	43	51	76	18	17	0
2	37	49	49	54	54	55	76	87	9	11	0
3	49	67	67	71	71	73	87	97	13	16	0

Total elapsed Time = 97 hours

Idle Time for Machine 1 = $97 - 61 = 36$ hours

Idle Time for Machine 2 = $97 - 61 + 57 = 93$ hours

Idle time for Machine 3 = $97 - 73 + 63 = 87$ hours

Idle Time for Machine 4 = 23 hours

1220) When passing is not allowed, solve the sequencing problem giving an optimal solution.

Job	machines (processing time in hours)			
	M_1	M_2	M_3	M_4
A	11	4	2	11
B	8	1	8	8
C	12	2	3	12
D	13	2	2	13

soln:-

$$\text{Min of } M_1 = 8$$

$$\text{Min of } M_4 = 8$$

$$\text{max of } (M_2, M_3) = 8$$

$$\text{Min of } M_1 \geq \text{max of } (M_2, M_3)$$

$$\text{Min of } M_4 \geq \text{max of } (M_2, M_3)$$

$$G = M_1 + M_2 + M_3 ; H = M_2 + M_3 + M_4$$

Job	A	B	C	D
G	17	17	17	17
H	17	17	17	17

A	B	C	D
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Job	M-1		M-2		M-3		M-4		Idle Time for M-2	Idle Time for M-3	Idle Time for M-4
	In	Out	In	Out	In	Out	In	Out			
A	0	11	11	15	15	17	17	28	11	15	17
B	11	19	19	20	20	28	28	36	4	3	0
C	19	31	31	33	33	36	36	48	11	5	0
D	31	44	44	46	46	48	48	61	11	10	0

Total elapsed time = 61 hours

Idle time for M-1 = $61 - 44 = 17$ hours

Idle time for M-2 = $61 - 46 + 37 = 52$ hours

Idle time for M-3 = $61 - 48 + 33 = 46$ hours

Idle time for M-4 = 17 hours.

1216) a) A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The times required to perform the printing and binding operation for each book are known. Determine the order in which the books should be processed in order to minimize the total time required to process all the books. Find also the total time required (clearly state an algorithm you might use)

Book	1	2	3	4	5
Printing time	40	90	80	60	50
Binding time	50	60	20	30	40

Printing time	40	90	80	60	50
Binding time	50	60	20	30	40

1	2	5	4	3
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Book	Printing time		Binding time		Idle time for Binding time
	In	Out	In	Out	
1	0	40	40	90	40
2	40	130	130	190	40
5	130	180	190	230	0
4	180	240	240	270	10
3	240	320	320	340	50

Total elapsed time = 5 hours 40 minutes

Idle time for machine A = 20 minutes

Idle time for machine B = 2 hours 20 minutes

- b) Suppose that an additional operation is added to the process described in (a) viz, finishing. The times required for operations are given below.

Book	Finishing time (in minutes)				
	1	2	3	4	5
Finishing time	80	100	60	70	100

What is the order in which the books should be processed? Find also the minimum

total elapsed time

soln:

Min of printing time = 40

Min of finishing time = 60

max of Binding time = 60

Min of Finishing time $\geq$ Max of Binding time

$G = \text{Printing time} + \text{Binding time}$

$H = \text{Binding time} + \text{Finishing time}$

Book	1	2	3	4	5
G	90	150	100	90	90
H	130	160	80	100	140

5	1	4	2	3
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Book	Printing time		Binding time		Finishing time		Idle time for Binding	Idle time for Finishing
	In	Out	In	Out	In	Out		
5	0	50	50	90	90	190	50	90
1	50	90	90	140	190	270	0	0
4	90	150	150	180	270	340	10	0
2	150	240	240	300	340	440	60	0
3	240	320	320	340	440	500	20	0

Total elapsed time = 9 hours 20 minutes

Idle time for Printing time = $500 - 120$
= 180 minutes
= 3 hours

Idle time for Binding time = $500 - 340 + 140$
= 300 minutes
= 5 hours

Idle time for Finishing time = 90 minutes
= 1 hour 30 minutes

1217) The garments manufacturer to process 7 items through two stages of production cutting and sewing. The time taken for each of them at the different stages are given below in appropriate

Item	1	2	3	4	5	6	7
Process time cutting	5	7	3	4	6	7	12
sewing	2	6	7	5	9	5	8

a) Find an order in which these items are to be processed through these stages so as to minimize the processing time

b) Suppose a third stage of production is added viz. Pressing and packing with processing time for items as follows.

Item	1	2	3	4	5	6	7
Processing time (Pressing and packing)	10	12	11	13	12	10	11

Find an order in which these seven items are to be processed so as to minimize the time taken to process the items through all the three stages.

soln:

a)

Item	1	2	3	4	5	6	7
Cutting	5	7	3	4	6	7	12
Sewing	2	6	7	5	9	5	8

3	4	5	7	2	6	1
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Item	Cutting		Sewing		Idle time for Sewing
	In	Out	In	Out	
3	0	3	3	10	3
4	3	7	10	15	0
5	7	13	15	24	0
7	13	25	25	33	1
2	25	32	33	39	0
6	32	39	39	44	0
1	39	44	44	46	0

Total elapsed time = 46 hours

Idle time for ~~max~~ cutting = $46 - 44 =$

2 hours

Idle time for sewing = 4 hours.

b) Min of Cutting = 3
 Min of Pressing and packing = 10
 max of Sewing = 9

Min of Pressing and
 Packing $\geq$ max of sewing

$G = \text{Cutting} + \text{Sewing}$; $H = \text{Sewing} + \text{packing}$

Item	1	2	3	4	5	6	7
G	7	13	10	9	15	12	20
H	12	18	18	18	21	15	19

1	4	3	6	2	5	7
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Item	Cutting		Sewing		Packing		Idle time for cutting	Idle time for Packing
	In	Out	In	Out	In	Out		
1	0	5	5	7	7	17	5	7
4	5	9	9	14	17	30	2	0
3	9	12	14	21	30	41	0	0
6	12	19	21	26	41	51	0	0
2	19	26	26	32	51	63	0	0
5	26	32	32	41	63	75	0	0
7	32	44	44	52	75	86	3	0

Total elapsed time = 86 hours

Idle time for cutting = $86 - 44 = 42$ hours

Idle time for Sewing = $86 - 52 + 10 = 44$ hours

Idle time for Packing = 7 hours